Published by Institute of Physics Publishing for SISSA

Received: *December 28, 2006*
Revised: *January 27, 2007*
Accepted: *April 18, 2007*
Published: *April 30, 2007*

Localization of fermions on a string-like defect

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Abstract: We study localization of bulk fermions on a string-like defect with the exponentially decreasing warp factor in six dimensions with inclusion of U(1) gauge background from the viewpoint of field theory, and give the conditions under which localized spin 1/2 and 3/2 fermions can be obtained.

Keywords: Large Extra Dimensions, Field Theories in Higher Dimensions.

Contents

1. Introduction

Recently, there has been considerable activity in the study of models that involve new extra dimensions. The possible existence of such dimensions got strong motivation from theories that try to incorporate gravity and gauge interactions in a unique scheme, in a reliable manner. The idea dates back to the 1920's, to the works of Kaluza and Klein [1] who tried to unify electromagnetism with Einstein gravity by assuming that the photon originates from the fifth component of the metric.

Suggestions that extra dimensions may not be compact [2]–[6] or large [7, 8] can provide new insights for a solution of gauge hierarchy problem [8], of cosmological constant problem [3, 5, 9], and give new possibilities for model building. In ref. [6], an alternative scenario of the compactification has been put forward. This new idea is based on the possibility that our world is a three brane embedded in a higher dimensional space-time with non-factorizable warped geometry. In this scenario, we are free from the moduli stabilization problem in the sense that the internal manifold is noncompact and does not need to be compactified to the Planck scale any more, which is one of reasons why this new compactification scenario has attracted so much attention. An important ingredient of this scenario is that all the matter fields are thought of as confined to a 3-brane, whereas gravity is free to propagate in the extra dimensions.

Following the brane world models proposed by Randall and Sundrum [6], a fair amount of activity has been generated involving possible extensions and generalizations, among which, co-dimension two models in six dimensions have been a topic of increasing interest [10 − 13]. A useful review on topological defects in higher dimensional models and its relation to braneworlds is available in [14]. Apart from model construction, the question of solving the cosmological constant problem has been the primary issue addressed in several articles [15]. Other aspects such as cosmology, brane gravity [16], fermion families and

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chirality [17] etc. have been discussed by numerous authors. A list of some recent articles on codimension two models is provided in [18].

It is well-known by now that in the braneworld scenario it is necessary to introduce dynamics which can determine the location of the branes in the bulk. Ever since Goldberger and Wise [19] added a bulk scalar field to fix the location of the branes in five dimensions, investigations with bulk fields became an active area of research. It has been shown that the graviton [6] and the massless scalar field [20] have normalizable zero modes on branes of different types, that the Abelian vector fields are not localized in the Randall-Sundrum (RS) model in five dimensions but can be localized in some higher-dimensional generalizations of it [12]. In contrast, in [20, 21] it was shown that fermions do not have normalizable zero modes in five dimensions, while in [12] the same result was derived for a compactification on a string [11] in six dimensions. Subsequently, Randjbar-Daemi *et al* studied localization of bulk fermions on a brane with inclusion of scalar backgrounds [22] and minimal gauged supergravity [23] in higher dimensions and gave the conditions under which localized chiral fermions can be obtained.

Since spin half fields can not be localized on the brane [6, 12] in five or six dimensions by gravitational interaction only, it becomes necessary to introduce additional non-gravitational interactions to get spinor fields confined to the brane or string-like defect. The aim of the present article is to study localization of bulk fermions on a string-like defect with codimension 2 in U(1) gauge background. The solutions to Einstein's equations in two extra dimensions have been studied by many groups [9−11, 24, 25]. In this article, we first review the solutions with a warp factor in a general space-time dimension. Then, we shall prove that spin 1/2 and 3/2 fields can be localized on a defect with the exponentially decreasing warp factor if gauge and gravitational backgrounds are considered.

2. A string-like defect

Let us start with a brief review of a string-like defect solution to Einstein's equations with sources. We consider Einstein's equations with a bulk cosmological constant Λ and an energy-momentum tensor T_{MN} in general D dimensions:

$$R_{MN} - \frac{1}{2} g_{MN} R = -\Lambda g_{MN} + \kappa_D^2 T_{MN}, \tag{2.1}$$

where κ_D denotes the D-dimensional gravitational constant with a relation $\kappa_D^2 = 8\pi G_N = 8\pi / M_*^{D-2}$, G_N and M_* being the D-dimensional Newton constant and the D-dimensional Planck mass scale, respectively, the energy-momentum tensor is defined as

$$T_{MN} = -\frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{MN}} \int d^D x \sqrt{-g} L_m. \tag{2.2}$$

Throughout this article we follow the standard conventions and notations of the textbook of Misner, Thorne and Wheeler [26].

We shall consider $D = (D_1 + D_2 + 1)$-dimensional manifolds with the geometry

$$\begin{aligned} ds^2 &= g_{MN} dx^M dx^N \\ &= \mathrm{e}^{-A(r)} \hat{g}_{\mu\nu}(x) dx^\mu dx^\nu + \mathrm{e}^{-B(r)} \tilde{g}_{ab}(y) dy^a dy^b + dr^2, \end{aligned} \tag{2.3}$$

where M, N denote D-dimensional space-time indices, $\mu, \nu = 0, 1, \ldots, D_1 - 1$, $a, b = 1, \ldots, D_2$, and the coordinates y^a cover an internal manifold K with the metric $\tilde{g}_{ab}(y)$. Moreover, we shall adopt the ansatz for the energy-momentum tensor respecting the spherical symmetry:

$$T^\mu_\nu = \delta^\mu_\nu t_1(r), \quad T^a_b = \delta^a_b t_2(r), \quad T^r_r = t_3(r), \tag{2.4}$$

where $t_i (i = 1, 2, 3)$ are functions of only the radial coordinate r.

Under these ansatzs, Einstein's equations (2.1) and the conservation law for energy-momentum tensor $\nabla^M T_{MN} = 0$ reduce to

$$\mathrm{e}^A \hat{R} + \mathrm{e}^B \tilde{R} - \frac{1}{4} D_1 (D_1 - 1)(A')^2 - \frac{1}{4} D_2 (D_2 - 1)(B')^2 \\ - \frac{1}{2} D_1 D_2 A' B' - 2\Lambda + 2\kappa_D^2 t_3 = 0, \tag{2.5}$$

$$\mathrm{e}^A \hat{R} + \frac{D_2 - 2}{D_2} \mathrm{e}^B \tilde{R} + D_1 A'' + (D_2 - 1) B'' - \frac{1}{2} D_1 (D_2 - 1) A' B' \\ - \frac{1}{4} D_1 (D_1 + 1)(A')^2 - \frac{1}{4} D_2 (D_2 + 1)(B')^2 - 2\Lambda + 2\kappa_D^2 t_2 = 0, \tag{2.6}$$

$$\mathrm{e}^B \tilde{R} + \frac{D_1 - 2}{D_1} \mathrm{e}^A \hat{R} + D_2 B'' + (D_1 - 1) A'' - \frac{1}{2} D_2 (D_1 - 1) A' B' \\ - \frac{1}{4} D_1 (D_1 + 1)(A')^2 - \frac{1}{4} D_2 (D_2 + 1)(B')^2 - 2\Lambda + 2\kappa_D^2 t_1 = 0, \tag{2.7}$$

$$t_3' = \frac{1}{2} D_1 A' (t_3 - t_1) + \frac{1}{2} D_2 B' (t_3 - t_2), \tag{2.8}$$

where $\hat{R}$ and $\tilde{R}$ are the scalar curvatures associated with the metric $\hat{g}_{\mu\nu}$ and $\tilde{g}_{ab}$, respectively, and the prime denotes the derivative with respect to r. Here we define the cosmological constant $\hat{\Lambda}$ on the $(D_1 - 1)$-brane by the equation

$$\hat{R}_{\mu\nu} - \frac{1}{2} \hat{g}_{\mu\nu} \hat{R} = -\hat{\Lambda} \hat{g}_{\mu\nu}. \tag{2.9}$$

It is now known that there are many interesting solutions to these equations (see, for instance, [24]). Here, we shall confine ourselves to the brane solutions with a warp factor

$$A(r) = cr, \tag{2.10}$$

where c is a constant.

If K is taken as a D_2-torus, then we have $\tilde{R} = 0$, and the general solutions with the warp factor (2.10) can be found as follows:

$$ds^2 = \mathrm{e}^{-cr} \hat{g}_{\mu\nu} dx^\mu dx^\nu + dr^2 + R_0^2 \, \mathrm{e}^{-B(r)} \delta_{ij} d\theta^i d\theta^j, \tag{2.11}$$

where

$$B(r) = cr + \frac{4}{D_1 c} \kappa_D^2 \int^r dr (t_3 - t_2), \tag{2.12}$$

$$c^2 = \frac{1}{D_1 (D_1 + 1)} (-8\Lambda + 8\kappa_D^2 \alpha), \tag{2.13}$$

$$\hat{R} = \frac{2D_1}{D_1 - 2} \hat{\Lambda} = -2\kappa_D^2 \beta. \tag{2.14}$$

Here t_2 takes the following form

$$t_2 = \alpha + \beta e^{cr}, \tag{2.15}$$

with α and β being some constants. Moreover, in order to guarantee the positivity of c^2, α should satisfy an inequality $-8\Lambda + 8\kappa_D^2 \alpha > 0$.

If K is taken as a unit D_2-sphere, then we have

$$\begin{aligned} d\Omega_{D_2}^2 &= \tilde{g}_{ab}(y) dy^a dy^b \\ &= d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2 + \cdots + \prod_{i=1}^{D_2-1} \sin^2\theta_i d\theta_{D_2}^2 . \end{aligned} \tag{2.16}$$

In the case of $D_2 = 1$, we have $\tilde{R} = 0$ and the solutions are the same as those of (2.11) [12]. For $D_2 \geq 2$ the solution with the warp factor (2.10) is of the form [12]

$$ds^2 = e^{-cr} \hat{g}_{\mu\nu} dx^\mu dx^\nu + dr^2 + R_0^2 d\Omega_{D_2}^2, \tag{2.17}$$

where

$$c^2 = \frac{-8\Lambda}{D_1(D_1 + D_2 - 1)}, \tag{2.18}$$

$$\hat{R} = \frac{2D_1}{D_1 - 2} \tilde{\Lambda} = 0, \tag{2.19}$$

here the sources satisfy the relations, $t_3 + D_2 t_2 - (D_2 - 1)t_1 = 0$ and $t_3 = t_1 = constant$, which are nothing but the relations satisfied in the spontaneous symmetry breakdown [24].

It is useful to consider a special case of the above general solutions (2.11) with $D_2 = 1$. A specific solution occurs when we have the spontaneous symmetry breakdown $t_3 = -t_2$ [24]:

$$ds^2 = e^{-cr} \hat{g}_{\mu\nu} dx^\mu dx^\nu + dr^2 + R_0^2 \, e^{-c_1 r} d\theta^2, \tag{2.20}$$

where

$$c^2 = \frac{1}{D_1(D_1 + 1)} (-8\Lambda + 8\kappa_D^2 t_2) > 0, \tag{2.21}$$

$$c_1 = c - \frac{8}{D_1 c} \kappa_D^2 t_2, \tag{2.22}$$

$$\hat{R} = \frac{2D_1}{D_1 - 2} \hat{\Lambda} = 0. \tag{2.23}$$

This special solution would be utilized to analyse localization of fermionic fields on a string-like defect in the next section.

3. Localization of fermions

In this section, for clarity we shall limit our attention to a specific string-like solution (2.20) as well as $D = 6$ since the generalization to the general solutions (2.11) is straightforward.

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In this paper, we have the physical setup in mind such that "local cosmic string" sits at the origin $r = 0$ and then ask the question of whether various bulk fermions with spin 1/2 and 3/2 can be localized on the brane with the exponentially decreasing warp factor by means of the gravitational interaction and gauge background. Of course, we have implicitly assumed that various bulk fields considered below make little contribution to the bulk energy so that the solution (2.20) remains valid even in the presence of bulk fields.

3.1 Spin 1/2 fermionic field

In this subsection we study localization of a spin 1/2 fermionic field in gravity (2.20) and gauge backgrounds. It will be shown that provided that the gauge field A_r satisfies certain condition, there is a localized zero mode on the string-like defect.

Let us consider the Dirac action of a massless spin 1/2 fermion coupled to gravity and gauge field:

$$S_m = \int d^D x \sqrt{-g} \bar{\Psi} i \Gamma^M D_M \Psi, \tag{3.1}$$

from which the equation of motion is given by

$$\Gamma^M (\partial_M + \omega_M - ieA_M)\Psi = 0, \tag{3.2}$$

where $\omega_M = \frac{1}{4}\omega_M^{\bar{M}\bar{N}} \Gamma_{\bar{M}} \Gamma_{\bar{N}}$ is the spin connection with $\bar{M}, \bar{N}, \cdots$ denoting the local Lorentz indices, Γ^M and $\Gamma^{\bar{M}}$ are the curved gamma matrices and the flat gamma ones, respectively, and A_M is a U(1) gauge field. The RS model is the special case with $D_2 = 0$ and $A_M = 0$. From the formula $\Gamma^M = e^M_{\bar{M}} \Gamma^{\bar{M}}$ with $e^{\bar{M}}_M$ being the vielbein, we have the relations:

$$\Gamma^\mu = \mathrm{e}^{\frac{1}{2}cr} \hat{e}^\mu_{\bar{\mu}} \Gamma^{\bar{\mu}}, \quad \Gamma^r = \delta^r_{\bar{r}} \Gamma^{\bar{r}}, \quad \Gamma^\theta = R_0^{-1} \mathrm{e}^{\frac{1}{2}c_1 r} \delta^\theta_{\bar{\theta}} \Gamma^{\bar{\theta}}. \tag{3.3}$$

The spin connection $\omega_M^{\bar{M}\bar{N}}$ in the covariant derivative $D_M\Psi = (\partial_M + \frac{1}{4}\omega_M^{\bar{M}\bar{N}} \Gamma_{\bar{M}} \Gamma_{\bar{N}} - ieA_M)\Psi$ is defined as

$$\omega_M^{\bar{M}\bar{N}} = \frac{1}{2} e^{N\bar{M}} (\partial_M e_N^{\bar{N}} - \partial_N e_M^{\bar{N}}) - \frac{1}{2} e^{N\bar{N}} (\partial_M e_N^{\bar{M}} - \partial_N e_M^{\bar{M}}) - \frac{1}{2} e^{P\bar{M}} e^{Q\bar{N}} (\partial_P e_{Q\bar{R}} - \partial_Q e_{P\bar{R}}) e_M^{\bar{R}}. \tag{3.4}$$

So the non-vanishing components of ω_M are

$$\omega_\mu = \frac{1}{4} c \Gamma_r \Gamma_\mu + \hat{\omega}_\mu \,, \tag{3.5}$$

$$\omega_\theta = \frac{1}{4} c_1 \Gamma_r \Gamma_\theta \,, \tag{3.6}$$

where $\hat{\omega}_\mu = \frac{1}{4}\bar{\omega}_\mu^{\bar{\mu}\bar{\nu}} \Gamma_{\bar{\mu}} \Gamma_{\bar{\nu}}$ is the spin connection derived from the metric $\hat{g}_{\mu\nu}(x) = \hat{e}^{\bar{\mu}}_\mu \hat{e}^{\bar{\nu}}_\nu \eta_{\bar{\mu}\bar{\nu}}$.

Assume $A_\mu = A_\mu(x)$ and $A_{r,\theta} = A_{r,\theta}(r)$. The Dirac equation (3.2) then becomes

$$\left\{ \mathrm{e}^{\frac{1}{2}cr} \hat{e}^\mu_{\bar{\mu}} \Gamma^{\bar{\mu}} \hat{D}_\mu + \Gamma^r \left(\partial_r - c - \frac{1}{4} c_1 - ieA_r(r) \right) + \Gamma^\theta (\partial_\theta - ieA_\theta(r)) \right\} \Psi = 0, \tag{3.7}$$

where $\hat{e}^\mu_{\bar{\mu}} \Gamma^{\bar{\mu}} \hat{D}_\mu = \hat{e}^\mu_{\bar{\mu}} \Gamma^{\bar{\mu}} (\partial_\mu + \hat{\omega}_\mu - ieA_\mu)$ is the Dirac operator on the brane in the background of the gauge field A_μ. We are now ready to study the above Dirac equation for

6-dimensional fluctuations, and write it in terms of 4-dimensional effective fields. Since Ψ is a 6-dimensional Weyl spinor we can represent it by

$$\Psi = \begin{pmatrix} \Psi^{(4)} \\ 0 \end{pmatrix}, \tag{3.8}$$

where $\Psi^{(4)}$ is a 4-dimensional Dirac spinor. Our choice for the 6-dimensional constant gamma matrices $\Gamma^{\bar{M}}$, $M = 0, 1, 2, 3, \bar{r}, \bar{\theta}$ are

$$\Gamma^{\bar{\mu}} = \begin{pmatrix} 0 & \gamma^{\bar{\mu}} \\ \gamma^{\bar{\mu}} & 0 \end{pmatrix}, \quad \Gamma^{\bar{r}} = \begin{pmatrix} 0 & \gamma^{5} \\ \gamma^{5} & 0 \end{pmatrix}, \quad \Gamma^{\bar{\theta}} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tag{3.9}$$

where the $\gamma^{\bar{\mu}}$ are the 4-dimensional constant gamma matrices and γ^5 the 4-dimensional chirality matrix. Imposing the chirality condition $\gamma^5\Psi^{(4)} = +\Psi^{(4)}$, the Dirac equation (3.7) can be written as

$$\left\{ \mathrm{e}^{\frac{1}{2}cr}\hat{e}^{\mu}_{\bar{\mu}}\gamma^{\bar{\mu}}\hat{D}_{\mu} + \left(\partial_r - c - \frac{1}{4}c_1 - ieA_r(r)\right) + iR_0^{-1}\mathrm{e}^{\frac{1}{2}c_1 r}(\partial_\theta - ieA_\theta(r)) \right\} \Psi^{(4)} = 0. \tag{3.10}$$

Now, form the equation of motion (3.10), we will search for the solutions of the form

$$\Psi^{(4)}(x, r, \theta) = \psi(x)\alpha(r)\sum \mathrm{e}^{il\theta}, \tag{3.11}$$

where $\psi(x)$ satisfies the massless 4-dimensional Dirac equation $\hat{e}^{\mu}_{\bar{\mu}}\gamma^{\bar{\mu}}\hat{D}_{\mu}\psi = 0$. For s-wave solution, eq. (3.10) is reduced to

$$\left(\partial_r - c - \frac{1}{4}c_1 - ieA_r(r) + eR_0^{-1}\mathrm{e}^{\frac{1}{2}c_1 r}A_\theta(r)\right)\alpha(r) = 0. \tag{3.12}$$

The solution of this equation is given by

$$\alpha(r) \propto \exp\left\{ cr + \frac{1}{4}c_1 r + ie\int^r drA_r(r) - eR_0^{-1}\int^r dr\ \mathrm{e}^{\frac{1}{2}c_1 r}A_\theta(r) \right\}. \tag{3.13}$$

So the fermionic zero mode reads

$$\Psi \propto \begin{pmatrix} \psi \\ 0 \end{pmatrix} \exp\left\{ cr + \frac{1}{4}c_1 r + ie\int^r drA_r(r) - eR_0^{-1}\int^r dr\ \mathrm{e}^{\frac{1}{2}c_1 r}A_\theta(r) \right\}. \tag{3.14}$$

Now we wish to show that this zero mode is localized on the defect sitting around the origin $r = 0$ under certain conditions. The condition for having localized 4-dimensional fermionic field is that $\alpha(r)$ is normalizable. It is of importance to notice that normalizability of the ground state wave function is equivalent to the condition that the "coupling" constant is nonvanishing.

Substituting the zero mode (3.14) into the Dirac action (3.1), the effective Lagrangian for ψ then becomes

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \int drd\theta\sqrt{-g}\bar{\Psi}i\Gamma^M D_M\Psi \\ &= I_{1/2}\ \sqrt{-\hat{g}}\ \bar{\psi}i\hat{e}^{\mu}_{\bar{\mu}}\gamma^{\bar{\mu}}\hat{D}_{\mu}\psi, \end{aligned} \tag{3.15}$$

where

$$I_{1/2} \propto \int_0^\infty dr \exp\left(\frac{1}{2}cr - 2eR_0^{-1}\int^r dr\ \mathrm{e}^{\frac{1}{2}c_1 r}A_\theta(r)\right), \tag{3.16}$$

In order to localize spin 1/2 fermion in this framework, the integral (3.16) should be finite. When the gauge background vanishes, this integral is obviously divergent for $c > 0$ while it is finite for $c < 0$. This situation is the same as in the case of the domain wall in the RS framework [20] where for localization of spin 1/2 field additional localization method by Jackiw and Rebbi [27] was introduced. Now let us look for the condition for localization of spin 1/2 field. Obviously, the A_r gauge field doesn't contribute to the integral (3.16). The requirement that the integral (3.16) should be finite is easily satisfied. For example, a simple choice is

$$A_\theta(r) = \lambda \mathrm{e}^{-\frac{1}{2}c_1 r}, \tag{3.17}$$

where λ is a constant satisfying the condition

$$\lambda > \frac{c}{4e}R_0. \tag{3.18}$$

Another choice can be taken as the following form

$$A_\theta(r) = \mathrm{e}^{-\frac{1}{2}c_1 r}r^n \tag{3.19}$$

with $n \geq 1$, or the more special and interesting form

$$A_\theta(r) = \left(\frac{c}{4e}R_0 + r^n\right)\mathrm{e}^{-\frac{1}{2}c_1 r} \tag{3.20}$$

with $n \geq 0$. So spin 1/2 field is localized on a defect with the exponentially decreasing warp factor under condition (3.17) or (3.19) or (3.20). Of course, there are many other choices which result in finite $I_{1/2}$.

3.2 Spin 3/2 fermionic field

Next we turn to spin 3/2 field, in other words, the gravitino. Let us start by considering the action of the Rarita-Schwinger gravitino field:

$$S_m = \int d^D x\sqrt{-g}\bar{\Psi}_M i\Gamma^{[M}\Gamma^N\Gamma^{R]}D_N\Psi_R, \tag{3.21}$$

where the square bracket denotes the anti-symmetrization, and the covariant derivative is defined with the affine connection $\Gamma^R_{MN} = e^R_{\bar{M}}(\partial_M e^{\bar{M}}_N + \omega^{\bar{M}\bar{N}}_M e_{N\bar{N}})$ by

$$D_M\Psi_N = \partial_M\Psi_N - \Gamma^R_{MN}\Psi_R + \omega_M\Psi_N - ieA_M\Psi_N. \tag{3.22}$$

From the action (3.21), the equations of motion for the Rarita-Schwinger gravitino field are given by

$$\Gamma^{[M}\Gamma^N\Gamma^{R]}D_N\Psi_R = 0. \tag{3.23}$$

For simplicity, from now on we limit ourselves to the flat brane geometry $\hat{g}_{\mu\nu} = \eta_{\mu\nu}$. After taking the gauge condition $\Psi_r = \Psi_\theta = 0$, the non-vanishing components of the covariant derivative are calculated as follows:

$$D_\mu \Psi_\nu = \partial_\mu \Psi_\nu + \frac{1}{4} c \Gamma_r \Gamma_\mu \Psi_\nu - ieA_\mu \Psi_\nu, \tag{3.24}$$

$$D_\mu \Psi_r = \frac{1}{2} c \Psi_\mu, \tag{3.25}$$

$$D_r \Psi_\mu = \partial_r \Psi_\mu + \frac{1}{2} c \Psi_\mu - ieA_r \Psi_\mu, \tag{3.26}$$

$$D_\theta \Psi_\mu = \partial_\theta \Psi_\mu + \frac{1}{4} c_1 \Gamma_r \Gamma_\theta \Psi_\mu - ieA_\theta \Psi_\mu. \tag{3.27}$$

Again we assume $A_\mu = A_\mu(x)$ and $A_{r,\theta} = A_{r,\theta}(r)$, and represent Ψ_μ as the following form

$$\Psi_\mu = \begin{pmatrix} \Psi_\mu^{(4)} \\ 0 \end{pmatrix}, \tag{3.28}$$

where $\Psi_\mu^{(4)}$ is the 4D Rarita-Schwinger gravitino field.

Imposing the chirality condition $\gamma^5 \Psi_\mu^{(4)} = +\Psi_\mu^{(4)}$, and substituting eqs. (3.24)–(3.28) into the equations of motion (3.23), we will look for the solutions of the form

$$\Psi_\mu^{(4)}(x, r, \theta) = \psi_\mu(x) u(r) \sum \mathrm{e}^{il\theta}, \tag{3.29}$$

where $\psi_\mu(x)$ satisfies the following 4-dimensional equations $\gamma^\mu \psi_\mu = \partial^\mu \psi_\mu = \gamma^{[\mu}\gamma^\nu\gamma^{\rho]}(\partial_\nu - ieA_\nu)\psi_\rho = 0$. Then the equations of motion (3.23) reduce to

$$\left(\partial_r - \frac{1}{2}c - \frac{1}{4}c_1 - ieA_r(r) + eR_0^{-1} \mathrm{e}^{\frac{1}{2}c_1 r} A_\theta(r)\right) u(r) = 0, \tag{3.30}$$

form which $u(r)$ is easily solved to be

$$u(r) \propto \exp\left\{\frac{1}{2}cr + \frac{1}{4}c_1 r + ie\int^r dr A_r(r) - eR_0^{-1}\int^r dr\ \mathrm{e}^{\frac{1}{2}c_1 r} A_\theta(r)\right\}. \tag{3.31}$$

In the above we have considered the s-wave solution.

Let us substitute the zero mode (3.31) into the Rarita-Schwinger action (3.21). It turns out that the effective Lagrangian becomes

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \int dr d\theta \sqrt{-g}\, \bar{\Psi}_M i \Gamma^{[M}\Gamma^N\Gamma^{R]} D_N \Psi_R \\ &= I_{3/2}\ \bar{\psi}_\mu i \gamma^{[\mu}\gamma^\nu\gamma^{\rho]}(\partial_\nu - ieA_\nu)\psi_\rho. \end{aligned} \tag{3.32}$$

where the integral $I_{3/2}$ is defined as

$$I_{3/2} \propto \int_0^\infty dr \exp\left(\frac{1}{2}cr - eR_0^{-1}\int^r dr\ \mathrm{e}^{\frac{1}{2}c_1 r} A_\theta(r)\right). \tag{3.33}$$

In order to localize spin 3/2 fermion, the integral $I_{3/2}$ must be finite. But this expression is equivalent to $I_{1/2}$ up to an overall constant factor so we encounter the same result as in spin 1/2 field. This shows that the solution (3.31) is normalizable under the condition (3.17) or (3.19) or (3.20) for not only the exponentially increasing but also the exponentially decreasing warp factor.

4. Discussions

In this paper, we have investigated the possibility of localizing the spin 1/2 and 3/2 fermionic fields on a brane with the exponentially decreasing warp factor, which also localizes the graviton. We first give a brief review of a string-like defect solution to Einstein's equations with sources, then check localization of fermionic fields on such a string-like defect with the background of gauge field from the viewpoint of field theory. We find that there is a same solution for subspace $K = D_2$-torus with any D_2 and $K = D_2$-sphere with $D_2 \geq 2$. It has been found that spin 1/2 and 3/2 fields can be localized on a defect with the exponentially decreasing warp factor if gauge and gravitational backgrounds are considered.

Localizing the fermionic degrees of freedom on the brane or the defect requires us to introduce other interactions but gravity. Recently, Parameswaran *et al* study fluctuations about axisymmetric warped brane solutions in 6-Dimensional minimal gauged supergravity and proved that, not only gravity, but Standard Model fields could "feel" the extent of large extra dimensions, and still be described by an effective 4-Dimensional theory [23]. Moreover, there are some other backgrounds could be considered besides gauge field and supergravity [28], for example, vortex background [29]. The topological vortex (especially Abrikosov-Nielsen-Olesen vortex) coupled to fermions may lead to chiral fermionic zero modes [30]. Usually the number of the zero modes coincides with the topological number, that is, with the magnetic flux of the vortex. In future, we wish to extend the present work to the Abelian Higgs model.

Acknowledgments

It is a pleasure to thank Dr Shaofeng Wu for interesting discussions. This work was supported by the National Natural Science Foundation of the People's Republic of China and the Fundamental Research Fund for Physics and Mathematic of Lanzhou University.

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